

# **3rd Alps-Adriatic Inverse Problems Workshop 2023 (AAIP 2023)**

Wednesday 05 July 2023 - Friday 07 July 2023

Alpen-Adria-Universität Klagenfurt

## **Book of Abstracts**



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1

## Far field operator splitting and completion for inhomogeneous medium scattering

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We consider scattering of time-harmonic acoustic waves by an ensemble of compactly supported penetrable scattering objects in 2D.

These scattering objects are illuminated by an incident plane wave.

The resulting total wave is the superposition of incident and scattered wave and solves a scattering problem for the Helmholtz equation.

For guaranteeing uniqueness, the scattered wave must fulfill the Sommerfeld radiation condition at infinity.

In our consideration, measurements of the total wave are replaced by the corresponding far field operator.

This operator contains all information about the scattered wave far away from the scattering objects for all possible illumination directions.

We are interested in two inverse problems.

On the one hand, given a limited observation of this far field operator, we want to determine its missing part, which we refer to as operator completion problem.

‘Limited observation’ in this context means, that we do not have access to measurements for all illumination directions or that we cannot measure in all observation directions around the scattering objects.

On the other hand, given the far field operator for the ensemble of scattering objects, we want to determine the far field operators of the individual scattering objects.

This is what we refer to as operator splitting problem.

Multiple reflection effects cause, in contrast to the first problem, the nonlinearity of this second problem.

We characterize spaces containing the individual, for the two problems relevant components of the far field operator.

Operators in these spaces turn out to have a low rank and sparsity properties with respect to some known modulated Fourier frame.

Furthermore, this rank and frame can be determined under knowledge of the locations and sizes of the scatterer’s components.

In my talk I will suggest two reformulations of the inverse problems, a least squares norm formulation and a  $l^1 \times l^1$ -norm minimization, and appropriate algorithms for solving these formulations numerically.

Moreover, I will present stability results for these reconstructions and support them by numerical experiments.

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## Deautoconvolution in the two-dimensional case

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We will have a discussion on the reconstruction of a real function of two real variables over the unit square from observations of its autoconvolution on  $[0, 2]^2 \subset \mathbb{R}^2$  (full data case) or on  $[0, 1]^2$  (limited data case). In an  $L^2$ -setting, twofoldness and uniqueness assertions can be obtained for the deautoconvolution problem in 2D. Moreover, by means of an example, we will illustrate the ill-posedness and also the stable approximate solutions to the two-dimensional deautoconvolution problem obtained by Tikhonov-type regularization with different penalties.

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## Nonlinear impedance boundary conditions in inverse obstacle scattering

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Nonlinear impedance boundary conditions in acoustic scattering are used as a model for perfectly conducting objects coated with a thin layer of a nonlinear medium. We consider a scattering problem for the Helmholtz equation with a nonlinear impedance boundary condition of the form  $\partial u_{\partial\nu + ik\lambda u = g(\cdot, u)} \text{ on } \partial D$ , where  $\partial u$  denotes the unit normal vector,  $\lambda \in L^\infty(\partial D)$  is a complex-valued impedance function, and  $g : \partial D \times \mathbb{C} \rightarrow \mathbb{C}$  gives an additional nonlinear term with respect to the total field  $u$ . The contributed talk is devoted to the well-posedness of the direct problem, the determination of the domain derivative, and the inverse problem, which consists in reconstructing the shape of the scattering object from given far field data. Numerical results are presented relying on an all-at-once regularized Newton-type method based on the linearization of the forward problem and of the domain-to-far-field operator.

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## On solving the inverse problem of diffractive tensor tomography via a transport equation

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The author discusses a method for dynamic tensor field tomography, which involves recovering a tensor field from its longitudinal ray transform in an inhomogeneous medium. The refractive index of the medium generates the Riemannian metric in the domain, and the goal is to solve the inverse source problem for the associated transport equation. While there are many results for recovering tensor fields in a static Euclidean setting, there are few inversion formulas and algorithms for general Riemannian metrics and time-dependent tensor fields. Tensor field tomography is equivalent to an inverse source problem for a transport equation, with the ray transform as given boundary data. In the dynamic case, the same approach can be used. To ensure the forward mappings are well-defined, existence and uniqueness for the transport equations must be proven. Unfortunately, the bilinear forms of the weak formulations do not satisfy the coercivity condition, so viscosity solutions are used instead. The author provides numerical evidence that the viscosity solution solves the original transport equation when the viscosity term is zero. Additionally, numerical experiments for the static case are discussed. It turns out that the adjoint mapping can also be expressed as solution of a transport equation and be solved by the method of characteristics. Numerical results for the reconstruction of stationary fields are given.

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## Accelerating the Landweber method for the eikonal equation by a CNN

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The nonlinear eikonal equation results as a high frequency approximation of the Helmholtz equation, more generally, of the wave equation. We investigate the eikonal equation with respect to the theory of inverse problems in the context of terahertz tomography. We integrate neural networks in the Landweber iteration for the reconstruction of the refractive index  $n(x)$ ,  $x \in \Omega$ , of an object. To reduce the computing time in the reconstruction process, we substitute the forward operator  $F$  by a Convolutional Neural Network  $\varphi_\Theta$ , so that we obtain the Landweber step  $n_{i+1}^\delta = n_i^\delta - \omega F'(n_i^\delta)^*(\varphi_\Theta(n_i^\delta) - y^\delta)$ . In a second step, we save energy in the learning process of our network by generating a sparse forward operator. We add to the cost functional of the CNN a  $l1$ -regularization term  $\alpha R(\Theta) = \alpha \sum_{i=1}^L \|\Theta^{(i)}\|_1$ , where  $\alpha$  denotes a regularization parameter,  $L$  the amount of layers and  $\Theta^{(i)}$  the matrix of weights for the associated layer. We compare the normal Landweber method with the learned and sparse one. Numerical results will be presented.

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## On tensor-based supervised learning

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We present a novel approach to supervised learning tasks, which is based on a suitable approximate solution to Fredholm integral equations of the first kind. In particular, we concentrate on least-squares collocation, functional tensor networks and alternating ridge regression.

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## Pulse Wave Analysis in the Human Brain

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Cardiac pulsations in the human brain have recently garnered interest due to their potential involvement in the pathogenesis of neurodegenerative diseases. The (pulse) wave, which describes

the velocity of blood flow along an intracranial artery, consists of a forward (anterograde) and backward (retrograde, reflected) part, but the measurement usually consists of a superposition of these components. In this talk, we provide a mathematical framework for the inverse problem of estimating the pulse wave velocity as well as the forward and backward component of the pulse wave separately, using MRI measurements on the middle cerebral artery. Additionally, we provide an analysis of the problem, which is necessary for the application of a solution method based on an alternate direction approach. The proposed method's applicability is demonstrated through numerical experiments using simulation data.

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## Determination of nonlinear local Material Properties using an Inverse Scheme

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The precise knowledge of the material properties is of utmost importance for motor manufacturers to design and develop highly efficient machines. However, due to different manufacturing processes, these material properties can vary greatly locally and the assumption of homogenized global material parameters is no longer feasible for the development process. The goal of our research project is to precisely determine these local magnetic material properties using a combined approach of measurements, numerical simulations and the applications of inverse methods. In this work, we focus on the identification of the local nonlinear permeabilities of electrical sheets considering cutting edge effects. In doing so, the electrical sheets are divided into subregions, each assigned with a nonlinear magnetic material model. Furthermore, we generate the measured data by forward simulations solving the magnetic field for the magneto-static case by applying the finite element (FE) method and overlay these data with a Gaussian white noise. Based on the generated data, we apply our inverse scheme on the simulation model to determine the parameters of the nonlinear material model. To ensure solvability, a Tikhonov regularization with a prior information for the parameter is considered. The accuracy and convergency of our approach is investigated.

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## Early stopping of untrained neural networks

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In recent years new regularisation methods based on neural networks have shown promising performance for the solution of ill-posed problems, e.g., in imaging science. Due to the non-linearity of the networks, these methods often lack profound theoretical justification. In this talk we rigorously discuss convergence for an untrained convolutional network. Untrained networks are particularly attractive for applications, since they do not require any training data. Its regularising property is solely based on the architecture of the network. Because of this, appropriate early stopping is essential for the success of the method. We show that the discrepancy principle is an adequate method for early stopping here, as it yields minimax optimal convergence rates.



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## Stability estimates for a random inverse source problem

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We consider the inverse random source problem to determine the strength of a random acoustic sound source by correlation data generated from the observation of the pressure signal of the emitted time harmonic acoustic waves. This model can be applied to aeroacoustics where regularisation methods for the inverse source problem constitute the best approach to determine a sound source. After uniqueness has been recently proven for this problem, we now investigate the stability properties. This presentation hence contains as one of the main result a rigorous proof of a logarithmic stability estimate as well as logarithmic convergence rates for spectral regularisation methods applied to the inverse source problem. These two results are obtained by verifying a variational source condition by methods developed by Hohage and Weidling. Therefore, we establish stability estimates using geometrical optics solutions. In this talk we will present numerical experiments as well which suggest logarithmic convergence rates.

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## Wave phenomena governed by fractional Moore-Gibson-Thompson equations

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In acoustics, higher-order-in-time equations arise when taking into account a class of (fractional) thermal relaxation laws in the modeling of sound wave propagation. In this talk, we will discuss the analysis of initial boundary value problems for a family of such equations and determine the behavior of solutions as the relaxation time vanishes. The studied model can be viewed as a generalization of the well-established (fractional) Moore–Gibson–Thompson equation with three, in general nonlocal, convolution terms involving two different kernels. The interplay of these convolutions will influence the uniform analysis and the limiting procedure.

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## Inexact proximal Langevin sampling

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In order to solve tasks like uncertainty quantification or hypothesis tests in Bayesian imaging inverse problems that go beyond the computation of point estimates, we have to draw samples from the posterior distribution. For log-concave but usually high-dimensional posteriors, Markov chain

Monte Carlo methods based on time discretizations of Langevin diffusion are a popular tool. If the potential defining the distribution is non-smooth, as is the case for many relevant imaging problems, these discretizations are usually of an implicit form. This leads to Langevin sampling algorithms that require the evaluation of proximal operators, which is, for some of the potentials relevant in imaging problems, only possible approximately using an iterative scheme. We investigate the behaviour of a proximal Langevin algorithm under the presence of errors in the evaluation of the proximal mappings. We generalize existing non-asymptotic and asymptotic convergence results of the exact algorithm to our inexact setting and quantify the additional bias between the target and the algorithm's stationary distribution due to the errors. We show that the additional bias stays bounded for bounded errors and converges to zero for decaying errors in a strongly convex setting. We show numerical results where we apply the inexact algorithm to sample from the posterior of typical imaging inverse problems in which we can only approximate the proximal operator by an iterative scheme.

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## Ultrasound Aberration Correction for Layered Media

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Ultrasound diagnostics is an important, non-invasive examination method in modern medicine and the reconstruction of an observed object using its reflected ultrasound waves is an inverse problem of current scientific interest. With focused ultrasound waves, one can look deep inside the human body without causing harm. However, a crucial assumption in the theory of focused ultrasound imaging is that the sound speed in the observed medium is constant, which is not the case in most clinical applications. It is possible to incorporate different sound speeds into the model, but at the cost of significantly higher algorithmical and computational complexity, which makes them not applicable in clinical practice. In this talk, we present a mathematical framework for modelling the aberrations caused by a layered medium. Subsequently, by assuming the geometry of the observed object and the sound speed of its layers to be known, an aberration correction algorithm is discussed. In the usual setting of ultrasound tomography, these parameters are of course unknown and have to be calculated from the observed ultrasound data. But by analyzing the stability of this direct model, we can determine the necessary accuracy in an estimate for the unknown parameters and the resulting errors in the inverse problem of reconstruction. The effectiveness of the proposed method is shown through numerical simulation using the k-Wave toolbox for Matlab. This work is a collaboration with S. Hubmer (RICAM) and R. Ramlau (JKU).

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## Gradient descent-based algorithms for inverse problems in variable exponent Lebesgue spaces

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Variable exponent Lebesgue spaces  $\ell^{(p_n)}$  have been recently proved to be the appropriate functional framework to enforce pixel-adaptive regularisation in signal and image processing applications, combined with gradient descent (GD) or proximal GD strategies. Compared to standard Hilbert or Euclidean settings, however, the application of these algorithms in the Banach setting of  $\ell^{(p_n)}$  is not straightforward due to the lack of a closed-form expression and the non-separability property of the underlying norm. We propose the use of the associated separable modular function, instead of the norm, to define algorithms based on GD in  $\ell^{(p_n)}$  and consider a stochastic GD to reduce the per-iteration cost of iterative schemes, used to solve linear inverse real-world image reconstruction problems.

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## Multipoint formulas in inverse problems and their numerical implementation

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We present the first numerical study of multipoint formulas for finding leading coefficients in asymptotic expansions arising in potential and scattering theories. In particular, we implement different formulas for finding the Fourier transform of potential from the scattering amplitude at several high energies. We show that the aforementioned approach can be used for essential numerical improvements of classical results including the slowly convergent Born-Faddeev formula for inverse scattering at high energies. The approach of multipoint formulas can be also used for recovering the X-ray transform of potential from boundary values of the scattering wave functions at several high energies. Determination of total charge (electric or gravitational) from several exterior measurements is also considered. In addition, we show that the aforementioned multipoint formulas admit an efficient regularization for the case of random noise.

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## Recovering both the wave speed and the source function in a time-domain wave equation by injecting high contrast bubbles

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Dealing with the inverse source problem for the scalar wave equation, we have shown recently that we can reconstruct the spacetime dependent source function from the measurement of the wave, collected on a single point  $x$  and a large enough interval of time, generated by a small scaled bubble, enjoying large contrasts of its bulk modulus, injected inside the domain to image. Here, we extend this result to reconstruct not only the source function but also the variable wave speed. Indeed, from the measured waves, we first localize the internal values of the travel time function by looking at the behavior of this collected wave in terms of time. Then from the Eikonal equation, we recover the wave speed. Second, we recover the internal values of the wave generated only by the background (in the absence of the small particles) from the same measured data by inverting a Volterra integral operator of the second kind. From this reconstructed wave, we recover the source function at the expense of a numerical differentiation.

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## Acoustic Cavitation using Resonating Micro-Bubbles. Analysis in the Time-Domain

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We study the time-domain acoustic wave propagation in the presence of a micro-bubble. This micro-bubble is characterized by a mass density and bulk modulus which are both very small as compared to the ones of the uniform and homogeneous background medium. The goal is to estimate the amount of pressure that is created very near (at a distance proportional to the radius of the bubble) to the bubble. We show that, at that small distance, the dominating field is reminiscent to the wave created by a point-like obstacle modeled formally by a Dirac-like heterogeneity with support at the location of the bubble and the contrast between the bubble and background material as the scattering coefficient. As a conclusion, we can tune the bubbles material properties so that the pressure near it reaches a desired amount. Such design might be useful in the purpose of acoustic cavitation where one needs enough, but not too much, pressure to eliminate unwanted anomalies. The mathematical analysis is done using time-domain integral equations and asymptotic analysis techniques. A well known feature here is that the contrasting scales between the bubble and the background generate resonances (mainly the Minnaert one) in the time-harmonic regime. Such critical scales, and the generated resonances, are also reflected in the time-domain estimation of the acoustic wave. In particular, reaching the desired amount of pressure near the location of the bubble is possible only with such resonating bubbles.

**Key Words.** Time-Domain Acoustic Scattering, Contrasting Media, Bubbles, Asymptotic Analysis, Retarded Layer and Volume Potentials, Lippmann–Schwinger equation.

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## On the jump set of minimizers of scalar and vectorial Total-Variation based regularization problems

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It has been shown a long time ago that the discontinuity set of the solution of a denoising problem by total variation minimization ("Rudin-Osher-Fatemi") is a subset of the discontinuity set of the original data, if smooth enough. In this talk, I will review the techniques used in the scalar setting, a variant developed by T. Valkonen which in theory addresses more cases (including vectorial), and a much simpler approach, developed in collaboration with M. Łasica (Warsaw) and inspired by T. Valkonen's techniques, yet which extends his results to even more cases.

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## Uncertainty Quantification of Inclusion Boundaries

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Computed tomography (CT) imaging is the task of reconstructing a positive attenuation field (in the form of an image) from a finite number of projections (e.g., sinograms). CT reconstruction is often followed by an image segmentation step to partition the image into piecewise smooth/constant regions. The boundaries between such regions often carry valuable information.

In this talk, we will describe a Bayesian framework for reconstructing the boundaries of piecewise constant regions in the CT problem in an infinite-dimensional setting. Since the regularity of boundaries carries crucial information in many inverse problem applications, e.g., in medical imaging for identifying malignant tissues or in the analysis of electroencephalogram for epileptic patients, we characterize the regularity of the boundary by means of its fractional differentiability. The proposed Bayesian formulation has a hierarchical structure, which simultaneously estimates the boundary and its regularity. In addition, we quantify the uncertainties in the estimates.

Our approach is goal oriented, meaning that we directly detect the discontinuities from the data, instead of reconstructing the entire image. This drastically reduces the dimension of the problem, which makes the application of Markov Chain Monte Carlo (MCMC) methods feasible.

We will show that the proposed method provides an excellent platform for challenging X-ray CT scenarios (e.g., in case of noisy data, limited angle, or sparse angle imaging). Furthermore, this framework can be extended to reconstruct 2D surfaces, track the changes of the boundaries, and handle other types of noise.

This work has been published or submitted, see [1, 2].

(joint work with Babak M. Afkham, Nicolai A. Riis, Per Christian Hansen)

#### References

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## Multiharmonic expansions for nonlinearity identification in wave type equations

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We consider an undetermined coefficient inverse problem for a nonlinear partial differential equation occurring in high intensity ultrasound propagation as used in acoustic tomography.

In particular, we investigate the recovery of the nonlinearity coefficient commonly labeled as  $B/A$  in the literature, which is part of a space dependent coefficient  $\kappa$  in the Westervelt equation governing nonlinear acoustics.

Corresponding to the typical measurement setup, the overposed data consists of time trace measurements on some zero or one dimensional set  $\Sigma$  representing the receiving transducer array.

In this talk, we will show some recent results pertaining to the formulation of this problem in frequency domain and numerical reconstruction of piecewise constant coefficients in two space dimensions.

This is joint work with Bill Rundell, Texas A&M University.

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## Towards optimal sensor placement for sparse inverse problems

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In this talk, we study parameter identification problems from a finite number of measurements under a sparsity assumption. Since the data is contaminated by Gaussian noise, a statistical framework for its recovery is considered. It relies on two main ingredients, first, a convex but nonsmooth Tikhonov point estimator over the space of Radon measures and, second, a suitable mean-squared error based on its Hellinger-Kantorovich (H-K) distance to the ground truth.

Assuming standard non-degenerate source conditions as well as applying careful linearization arguments, we derive a sharp upper bound for the H-K distance between the aforementioned ground truth and an estimator. On the one hand, this allows to derive asymptotic convergence results for the mean-squared error, which is later used as a crucial tool for sensor placement problem. Finally, we present some numerical results to illustrate our theory.

This is a joint work with Konstantin Pieper and Daniel Walter.

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## The topological derivative as an imaging tool for object detection

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Detecting defects embedded in a medium is a problem of paramount interest in a variety of fields, including medical imaging, non-destructive testing of materials and geophysical exploration.

In this talk we present numerical methods based on topological derivative computations for the detection of multiple objects. The method provides an indicator function capable of classifying each point in the region of interest as belonging to the background medium or to an object, without any a priori assumption about the number, size, shape, or location of the objects.

The performance of the method in different applications, including acoustic, electromagnetic, and thermographic inspection will be shown.

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## Stability of Bayesian Optimal Experimental Design in Inverse Problems

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We study the stability properties of the expected utility function in Bayesian optimal experimental design. We provide a framework for this problem in the case of expected information gain criterion in an infinite-dimensional setting, where we obtain the convergence of the expected utility with respect to perturbations. To make the problem more concrete we demonstrate that non-linear

Bayesian inverse problems with Gaussian likelihood satisfy necessary assumptions in our theory. Some numerical simulations with different examples are explored.

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## Comparison of the Multiscale Hierarchical Decomposition Method and generalized Tikhonov regularization

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The Multiscale Hierarchical Decomposition Method (MHDM) is a popular method originating from mathematical imaging. In its original context, it is very well suited to recover approximations with fine details from blurred and noise-corrupted images. The main idea is to iteratively decompose an image into a cartoon and a texture part at different scales. We consider the algorithm in a more general framework, allowing one to apply it to a wider variety of problems. In this talk, we focus on comparing the MHDM to generalized Tikhonov regularization with seminorm regularizers. We propose a necessary and sufficient condition for the iterates of the MHDM to coincide with the minimizers of the Tikhonov regularization. We illustrate the result on finite dimensional  $\ell^1$  regularization and one-dimensional total variation denoising.

Joint work with Elena Resmerita and Stefan Kindermann.

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## A paraxial approach for the inverse problem of vibroacoustic imaging in frequency domain

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Vibroacoustic imaging by means of ultrasound is an imaging method that was developed to achieve higher resolutions by sending in high frequency waves and making use of the difference frequency to avoid the drawbacks of scattering and stronger attenuation at high frequencies. We make use of a paraxial approach for the directive beams and arrive at a system of PDEs that involves space dependent parameters. Reconstructing these parameters yields a spatial image of the region of interest. In this talk, we will deal with the modeling and inverse problem for vibroacoustic imaging.

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## Multiscale approaches for removing multiplicative noise

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This work adapts a variety of both classical and new multiplicative noise removing models to the multiscale hierarchical decomposition form, following the idea of a seminal paper from 2004. Well-definedness and convergence properties are provided for the proposed methods, as well as comprehensive numerical experiments and comparisons.

This is joint work with Joel Barnett, Wen Li and Luminita Vese.