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Presentation Title

Matching with Minimum Quotas
(with Szilvia Pápai)

Abstract

In this paper we consider a model to match agents on two sides of the market to each other, a matching model with minimum quotas for one side and with or without different types of agents. Each agent accepts all the agents on the other side but she has an individual preference ranking over the agents on the other side. Minimum quota requirements are common in many real-life matching problems. For instance, a minimum number of participants may be required to open a course or specialization, or to run a business, and a minimum quota maybe used to populate unpopular positions (such as rural hospital residencies). Minimum quotas may also be introduced with the aim of providing fair opportunities to agents who need help and protection. Imposing a minimum number of positions set aside for specific agent types is used as a tool to create and promote diversity which is both an every-day reality and an important objective in multicultural countries like Canada. Quotas that need to be filled minimally with specific types of agents who need protection (e.g. students from minority communities) are aimed at providing help and guaranteeing the balance of different types of agents at the same time. The main objective of this project is to find algorithms which respect minimum quotas and find the matchings which are both nonwasteful and fair if there exists such a matching. Otherwise the algorithms will find fair or nonwasteful matchings. We introduce a novel graph as a part of algorithms which is also a big assist to elaborate the matching problem and clearly represents the algorithm procedures. Compared to existing algorithms for achieving a matching which is fair, nonwasteful or both, our algorithms are integrated and more intuitive. Our algorithms, CNWF & FCNW (constrained nonwasteful fair & fair con-strained nonwasteful), start with finding the range of possible matchings when there are minimum quotas. Based on these possible matchings we construct a representative directed graph with feasible matched pairs as vertices. This graph contains two types of arcs, one pertaining to the preferences of one side of the market, and the other to the other side priorities. The graph provides the basis for selecting feasible matchings which satisfy minimum quotas and are both fair and nonwasteful, and if there do not exist such matchings, CNWF selects a constrained nonwasteful matching with maximum degree of fairness and FCNW selects a fair matching with maximum degree of nonwastefulness. Furthermore, we show that our algorithms work if the agents on one side of the market have different types. While existing applicable algorithms only partially deal with the issue of fairness versus nonwastefulness, we are able to find intuitive solutions, taking advantage of the clear structure of the graph, which summarizes exactly the information needed for making matching decisions in the presence of binding minimum quotas. This provide an ideal framework to explore the trade-offs and the general compatibility of the nonwastefulness and fairness notions, as well as their relations to incentive properties. We study the structure of the graph regarding different preferences of agents and minimum quotas in order to gain a thorough understanding of the structure of the problem, which allows us to create new mechanisms for matching in the presence of minimum quotas. Based on our findings and having studied numerous quantitative examples to understand the constructed graph, we prove that the graph can define fair and nonwasteful matchings, if such a matching exists. Furthermore, we characterize the specifics of the model and the preferences in terms of its representative graph, for which a fair and nonwasteful rule exists. This constitute a substantial improvement over current results and will integrate and unify the existing algorithms. We also, generalize these results to settings with multiple agent types, for which we use a family of representative directed graphs, and all this give us a new set of algorithms to choose from and a much better intuitive idea of what is possible for practical matching designers whose objective is to increase diversity and satisfy type-specific distributional constraints. In addition, we study the strategy-proofness of our algorithms in the presence of minimum quotas which is a common feature of many real-life matching problems. Then, we will introduce another algorithm, SPF (strategy-proof fair) which is strategy-proof and selects fair matchings based on our framework. This paper will hopefully serve as a foundation for future research which will generalize the newly designed algorithms to extend our findings to more complex matching models.

Keywords

Matching, Minimum Quotas, Fair, Nonwasteful, Strategy-proof

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